Structural Health Monitoring Activities at National Laboratories

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ABSTRACT

Sandia National Laboratories and Los Alamos National Laboratory have ongoing programs to assess damage in structures and mechanical systems from changes in their dynamic characteristics. This paper provides a summary of how both institutes became involved with this technology, their experience in this field and the directions that their research in this area will be taking in the future.

INTRODUCTION

At Sandia National Laboratories (SNL) in Albuquerque, New Mexico, and Los Alamos National Laboratory (LANL) in Los Alamos, New Mexico, teams of engineers with structural dynamics backgrounds have been studying the applicability of vibration-based damage identification (ID) methods to a variety of structural and mechanical systems. These engineers, whose experience is primarily in experimental modal analysis, were successful in competing for internal research funds (referred to as Laboratory Directed Research and Development (LDRD) funds) to begin investigations in this field. The vibration-based structural health monitoring efforts of both institutes that are described in this paper have their beginnings as LDRD projects. Both LDRD projects have been successful and follow-on funding has become available such that these efforts will be continuing at both institutes in the future.

SNL and LANL have many complimentary groups that provide an excellent base for the development of health monitoring technologies. At SNL, the joint work performed in structural system ID by the Modal Group of the Experimental Structural Dynamics Department and the Analysis Group of the Structural Dynamics Department have provided the framework for the health monitoring research that has occurred at SNL over the last three years. Similarly, work done by solid-state physicists in LANL's Condensed Matter and Thermal Physics Group has been coupled with work from the Engineering Analysis Group and the Measurement Technology Group to enhance the technology of vibration-based damage detection. In addition,

the close proximity (organizationally) of SNL's Aging Aircraft Project Department and the Non-Destructive Evaluation Department (NDE group) as well as SNL's Modal group's long term support of the Wind Energy Technology Department (Wind Energy Group) have also contributed to the work in damage detection. SNL's and LANL's contributions have been further strengthened by the close technological relationship that exists between the two institutes.

SUMMARY OF VIBRATION BASED HEALTH MONITORING ACTIVITIES AT SANDIA NATIONAL LABORATORIES

At SNL initial exposure with the health monitoring field was made by a joint delegation from the Analysis and Modal groups. They attended the NASA/Air Force System ID and Health Monitoring Workshop in 1990 from which the first health monitoring proposal was developed. Building upon the initial interest in the field, contact was made with NASA and the American Association of Railroads (AAR). Discussions with the AAR brought to light the problems with the many aging bridges in this country. Discussions were also initiated with the offshore oil industry.

As SNL became more involved in health monitoring, it became apparent that collaboration with universities could foster beneficial relationships. The first university interaction resulted in a joint proposal to the FAA from SNL and Prof. Norris Stubbs at Texas A&M University. Also, early interactions and support from Prof. Ken White's group at New Mexico State University allowed SNL to participate in the ground-breaking Rio Grande Interstate 40 (I-40) Bridge test.

Contact with Virginia Polytechnic Institute (VPI) through Prof. Robert West has provided SNL with state-of-the-art Laser Doppler Vibrometer (LDV) algorithms. The LDV has been used by SNL in a variety of applications including aircraft panels, composite plates, and, most recently, an outdoor test of a wind turbine blade. Collaboration with University of Texas at El Paso (UTEP) through Carlos Ferregut and Roberto Osegueda has further strengthened the understanding of LDV applications to health monitoring problems. This collaboration has produced a significant amount of the nation's experimental activity in this area. Collaborative efforts with Lee Peterson and K.C. Park of the University of Colorado at Boulder (CU) have developed new approaches to analyze and interpret data sets with both high spatial density and high modal density, such as those produced by the LDV. Also, this work produced new damage detection approaches based on structural flexibility matrix estimation and disassembly.

Collaboration with David Zimmerman at the University of Houston was initiated by the Analysis group covering system ID topics. This relationship was later expanded to provide SNL with expertise in the area of Model-Based Damage Detection. Model-based damage detection uses a reference or healthy model of the monitored system which is compared to a set of current measurements. If the model does not simulate the current measurements sufficiently, the system has changed and is assumed to be damaged. And most recently, the use of transmittance functions to detect small damage on relatively large structures has been investigated by Professors Schulz and Pai at North Carolina A&T in collaboration with SNL.

Collaboration was also established with other Government organizations. SNL's relationship with the National Renewable Energy Laboratory (NREL) has provided much motivation and research in the area of monitoring of wind turbine blades. A significant advance in SNL's involvement in Health Monitoring occurred in 1991 when the Federal Aviation Administration (FAA) established the Airworthiness Assurance and NDI Validation Center (AANC). To support this center, an aircraft hangar has been set up with a series of hardware specimens including complete transport and commuter aircraft. The facility replicates a working maintenance environment by incorporating both the challenges of physical inspection as well as the environmental factors which influence inspection reliability. Informal interactions with NASA Johnson Space Center have also been developed through the SNL Special Leave of Absence Program which has allowed one of the authors to work with NASA through a University of Houston program covering model correlation and structural health monitoring topics.

SUMMARY OF VIBRATION BASED HEALTH MONITORING ACTIVITIES AT LOS ALAMOS NATIONAL LABORATORY

Vibration-based damage detection work at LANL had its beginnings almost 15 years ago when engineers in the Advanced Engineering Technology Group attempted to identify the onset of seismically-induced buckling in scale model nuclear reactor containment structures from changes in their measured vibration response. This work was followed by attempts to infer damage in seismically loaded scale-model reinforced concrete shear wall structures from changes in their vibration response.

As a result of an LDRD-funded investigation, physicist in LANL's Condensed Matter and Thermal Physics Group developed and patented a damage ID system referred to as Resonant Ultrasound Spectroscopy (RUS) in the early 1990s. [1] This system combined sine-sweep vibration testing with a homodyne detection system to make very precise measurements of the resonant frequency of small test specimens. For objects of very regular geometry, such as ball bearings, this test system can provide very accurate indications of material or geometric anomalies, such as out-of-roundness of a ball bearing. Subsequent applications of RUS include the detection of salmonella poisoning in eggs from changes in their vibration characteristics, the screening of captured Gulf-War ammunition to determine if artillery shells contain conventional or chemical warheads, and the detection of cracks in machined parts.

Through collaboration with these physicists, engineers from the Advanced Engineering Technology Group (recently re-named the Engineering Analysis Group) were asked to be a primary participant in the damage ID tests on the I-40 Bridge over the Rio Grande.[2] These tests were performed in conjunction with engineers from SNL, faculty and students from New Mexico State University, and the New Mexico State Highway and Transportation Department. The engineers from LANL performed the experimental modal analyses of the bridge in its undamaged and damaged conditions while engineers from SNL ran a hydraulic shaker that provided the

input for these vibration tests. Subsequently, data from these tests has been made available to many universities. The physicists contributed to these tests by demonstrating a non-contact vibration measurement system based on a microwave interferometer designed and constructed at LANL.

Participation on this project lead to two additional ongoing LDRD-funded projects in LANL's Engineering Analysis Group related to vibration-based damage detection. Products resulting from these projects include an extensive review of the literature of vibration-based damage ID [3], a workshop held at LANL in September 1995 [4], and a MATLAB-based computer code known as DIAMOND [5] for statistically enhanced modal analysis, damage detection, and finite element model refinement.

OUTCOME OF SANDIA NATIONAL LABORATORIES' LDRD (SUCCESSES AND FOLLOW-ON WORK)

Most of SNL's technological and collaborative efforts have been funded by the LDRD. The one million dollar project spread over two years was entitled "Development of Structural Health Monitoring Techniques Using Dynamic Testing." This project was leveraged with funds from other LDRD projects covering System ID, Virtual Testing, Rapid Prototyping, as well as some LANL projects. Leveraging both these efforts and strong collaborations with private companies such as Holographics, Inc. and Flowind, Inc., a significant number of experimental and analytical sub-projects were performed. In fact, this LDRD supported the efforts of 25 individuals working on over 20 sub-projects and interacting with over 30 internal and external collaborators. Reference [6] contains a complete description of the LDRD efforts. Follow-on projects have been funded on traditional DOE structures, Air Force owned radar antennas, architectural surety, glass response to blast environments, and resonant fatigue testing of adhesive bonds. A field test of wind turbine health monitoring technologies has also been conducted.

APPLICATIONS

In the context of health monitoring, many structures have been studied by SNL. The majority of the work has focused on three main areas: aging aircraft, bridges, and wind turbines. In addition to modal analysis, many different technologies have been used to monitor the health of these structures. The LDV has proven to be valuable as a non-contacting measurement device. The LDV is capable of collecting data at more measurement points than traditional instruments such as accelerometers. The LDV is also proving itself in the field where a modal test can be performed without the usual mounting time of accelerometers. Among the other technologies used is the Natural Excitation Technique (NExT) [7] data reduction technique and neural networks as well as ultrasound and acoustic emission testing. Sub-projects devoted to understanding the utility of Electronic Speckle Pattern Interferometry (ESPI) were also performed.

Aging Aircraft

The aging aircraft facility at SNL has provided many opportunities to conduct health monitoring studies. Initially, work was done on a plate that simulated an aircraft skin. Initial studies on the plate utilized traditional modal techniques and simulated damage. Follow-on work consisted of the use of the LDV on the simulated skins to perform health monitoring studies.

A joint project was also initiated between SNL and a small company called Holographics, Inc. This collaboration was to extend the LDV technology from the initial studies. Frequency Response Functions (FRFs) were gathered from a McDonnell-Douglas DC9. The number of locations where data was gathered was 2233, which is an order of magnitude increase over traditional modal tests. Damage was induced by cutting a stringer, and a modal test was performed initially and after each cut. The data was also sent to CU for additional analysis. The size of the database prompted researchers at SNL and CU to develop procedures to condense the information into a usable set of important parameters.

Some work has also been performed on typical repairs seen on aircraft. A simple bolted repair joint was applied to an induced damaged stringer and the skin was scanned with the LDV. During this series of tests, five composite plates that simulated control surfaces and typical flaws seen in aircraft were also tested with the LDV. Using flexibility matrices and a technique known as flexibility disassembly [8], the flaws in the composite plates were located using the LDV data.

I-40 Bridge Work

SNL became involved with the I-40 bridge test through New Mexico State University, which was the lead institution for the experiment. They were interested in engaging SNL's expertise in modal analysis to develop a shaker excitation system for the bridge. SNL also provided accelerometers and information on the NExT procedure for this test. The NExT technique utilizes a background vibration source (such as automotive traffic or wind) as excitation for the test article. In the case of the I-40 bridge, traffic loading was used as the excitation source. From the response measurements resulting from this type of excitation, natural frequencies, damping, and mode shapes were extracted. SNL also acquired internal funds to support further involvement with the test.

The bridge work, in addition to strengthening the relationship between LANL and SNL, also produced significant advances in SNL's health monitoring technologies. A technique used for localizing errors in a finite element model, STRECH [9], was successfully adapted to be used in a health monitoring capacity. Also, a pseudo-model based damage detection algorithm (MAXCON) [10] was developed and proved useful in analyzing the I-40 bridge data.

Wind Energy

Much of SNL's efforts in health monitoring have been focused at wind energy applications. Structurally, the wind turbine is a fairly simple system. However, catastrophic failure of a working blade can mean damage to other blades, the tower, internal mechanical systems, other wind turbines, or to workers. Also, wind turbines see a tremendous number of fatigue cycles during a typical design lifetime. In conjunction with SNL's active Wind Energy group, many studies relating to failure of a wind turbine blade have been performed.

One of the first studies performed was a quasi-static fatigue test over a five month period. The structure was a Horizontal Axis Wind Turbine (HAWT) and the test was performed at the National Renewable Energy Laboratory (NREL). Modal tests were performed at various stages of the test. The test data included some unexpected phenomena. Following an initial drastic drop in all modal frequencies, most of the modal frequencies stayed constant until failure. At failure, most of the frequencies increased. Typically, frequencies will drop as a structure fails. Set-up dynamics and environmental conditions are believed to have caused the frequency shifts.

Two resonant fatigue tests were performed on Vertical Axis Wind Turbine (VAWT) blade sections. A resonant fatigue test exercises a structure at a natural frequency to decrease the time to failure. These tests also included periodic modal tests to establish a modal history of the fatiguing blades. In one of these tests the blade developed a span-wise crack that caused a shift in the torsional mode frequency while not affecting the frequency of the bending modes. This mode showed a monotonic decrease in frequency as a function of number of fatigue cycles.

A recent project involved the test of a wind turbine blade in the field with a LDV. The work included verifying that the LDV could take reliable data in the field, determining if NExT could be used with LDV data, and determining if simulated damage could be detected in NExT data. The analysis has not currently been completed.

Other Applications

Some other structures that have been studied from the perspective of health monitoring include nuclear power plants, tower guy anchors, and air compressor blades. These projects have been small efforts with little follow on work to date, but have greatly increased the national laboratories' experience base with real operating structures.

One of SNL's main mission is the surety of the nation's aging weapons systems. One weapon system application was an aerospace housing component [11]. Damage was simulated in the housing through a series of cuts in high strain regions. All damage cases were detected through the use of probabilistic neural networks. This highly successful project utilized information condensation techniques developed for use with the LDV.

Work is also being performed on the bearings of a large ground-based satellite dish. These systems are challenging due to the large size of the bearings and the slow, almost quasi-static rotation of the dishes. This work is currently in progress.

OUTCOME OF LOS ALAMOS NATIONAL LABORATORY'S LDRD (SUCCESSES AND FOLLOW-ON WORK)

The investigators at LANL can point to three primary successes that have resulted from the LDRD-funded investigations of vibration-base damage detection. First, the literature review that was recently published is, in the authors' opinion, the most comprehensive summary of the literature in this field to date.[3] Second, the computer code DIAMOND is to the authors' knowledge the only code that assembles many of the recent advances in vibration-based damage detection algorithms into one graphical user interface code.[5] When made available to the public, this code will allow researchers in this field to apply various linear and nonlinear algorithms to modal data to study the relative merits of the various damage ID methods. The code will allow researchers to easily modify existing algorithms and to add new algorithms as they are developed. Current plans are for the code to be distributed with sample data sets such as those from the I-40 bridge test so that researchers can apply their own algorithms to some standard data sets for comparative purposes. The final success of these projects relates to the fact that funding has been made available to continue this work with applications to our nuclear weapons systems. References [3,12-20] provide a summary of much of the work to date that has come from these LDRD projects.

APPLICATIONS

Most of the work conducted at LANL in the area of Structural Health Monitoring has focused on applications to highway bridges. Analysis of data sets from modal tests of bridges has demonstrated the importance of quantifying the variability of the measured modal parameters resulting from environmental conditions. Statistical analysis techniques such as Monte Carlo simulation and Bootstrap analysis have played an important role in the quantification of such variability effects, as well as the incorporation of these effects into various damage ID algorithms. Ongoing work is focused on the testing of idealized structures under laboratory conditions for the purposes of comparing the effectiveness and limitations of various damage ID techniques. Investigators at LANL have also been collaborating with Prof. Emin Aktan and Prof. Arthur Helmicki of the University of Cincinnati Infrastructure Institute to investigate modal parameter ID techniques in the context of structural damage ID [21]. Additionally, investigators from LANL are leading the compilation of a report on the State of the Art in Structural Identification of Constructed Facilities for the American Society of Civil Engineers.

To date, field verification of damage detection algorithms applied to large civil engineering structures are scarce as few full size structures are made available for such destructive testing. Because the I-40 bridges over the Rio Grande in Albuquerque, New Mexico were to be demolished and replaced, the investigators were able to introduce simulated cracks into the structure, perform vibration test before and after each level of damage had been introduced, and then use the test data to validate various damage ID methods. Staff from LANL and SNL performed experimental modal analyses on the bridge in its undamaged and damaged conditions. Researchers from Texas A&M University subsequently applied a damage detection algorithm to these data [22]. The same damage detection algorithm was independently applied by the LANL staff to these data and to numerical data from finite element simulations of the I-40 bridge where other damage scenarios were investigated. The data required by the damage ID algorithm are mode shapes and resonant frequencies for the damaged and undamaged bridge. Results from these investigations are some of the first comparative studies of various damage ID algorithms that have been reported in the technical literature [19].

Alamosa Canyon Bridge

The Alamosa Canyon Bridge in southern New Mexico has been designated as a bridge test facility by the New Mexico State Highway and Transportation Department. Numerous modal tests have been performed on this structure for the purposes of damage detection. With only limited abilities to introduce damage into this structure, recent tests have focused on quantifying the statistical variations in modal properties that result from changing environmental conditions. [15,16] It is imperative that these changes be quantified and that changes resulting from damage are shown to be either greater than or different from those resulting from the test-to-test variations. Recent tests have been performed with the intent of comparing different statistical analysis procedures.

Eight Degree of Freedom Test System

When reviewing the literature on vibration-based damage ID, most studies seem to examine either a beam or a very complex structure such as an offshore oil platform. In an attempt to provide data from some structures of intermediate complexity and at the same time trying to keep the cost of fabrication reasonable, an eight degree of freedom, lumped-mass system was designed such that small quantifiable changes in stiffness or mass can be easily incorporated into the system. In addition, nonlinearities such as a crack opening and closing, variable frictional damping mechanisms, and loose parts rattling around can also be simulated with this system. This system will be tested in a variety of configurations and results of these tests will be used for comparative studies of damage ID algorithms and finite element model updating methods.

Comparative Test Specimens

To provide data sets for comparative studies of various damage ID algorithms, students and engineers at LANL are conducting a series of tests on simple structures. The idea of these tests is to investigate a wide variety of structure types while keeping the cost of specimen fabrication low. These structures include an aluminum I-beam, an aluminum plate, a three-story "unistrut" frame structure, and a fifty-five gallon drum that is intended to simulate a shell structure. If time and funds permit, a composite plate and a reinforced concrete bridge pier will also be tested. Linear and nonlinear damage is being introduced incrementally in these structures and vibration data (both time-histories and frequency domain data) are being measured for the undamaged structure and after each damage state. Various damage ID methods will be applied to these data sets for comparative studies. It is the intent of this study that these data will be made available to other investigators for further investigation.

Weapons Systems

The primary follow-on work that has been generated by LANL's LDRD projects is the application of this technology to nuclear weapons systems. This application will form the mainstay of LANL's future vibration-based damage ID work. The complexity of these weapon systems will require the development of new data analysis techniques that rely more heavily on the identification of changes in nonlinear system response. It is anticipated that technological advancements made through the investigation of these systems will be directly applicable to civilian structures and mechanical systems as well.

WHERE ARE WE GOING FROM HERE?

The national laboratories are repositories of a great deal of the nation's expertise in applying new technologies to real operating structures, both military and civilian. The next steps in the laboratories' development of damage ID technology includes the formation and/or strengthening of collaborations involving technology creators at universities and users with real structures. The national laboratories play key roles in locating appropriate new technologies, developing them for field applications, and brokering these applications. The charter of the national labs also drives the need to attack problems of a national scale which are critical to the reliability, safety, and security of the American people. The decay of the infrastructure including civil, aerospace, industrial, communication, energy, and defense areas represents a problem requiring the attention of the national laboratories.

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